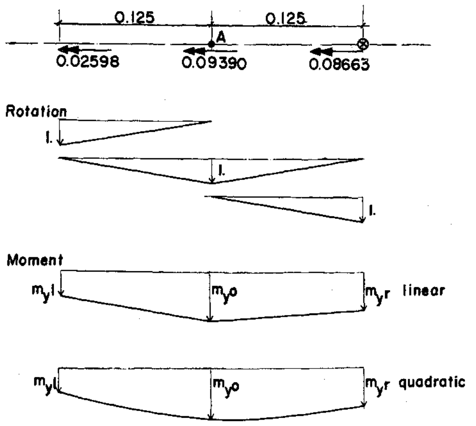

Fig. 3 Assumed distribution of rotation and of moment, equivalenced to concentrated nodal forces (moments) shown at top.

responding to the concentrated moments. Setting the integrals equal to the concentrated moments leads to a system of three equations. For the linear and quadratic assumptions of m_y, one can calculate $m_y o$, which is the value at A, as 0.6015 and 0.6764, respectively. It follows that all 3 values determined by using the nodal forces are less accurate than the approximate structural averaging procedure avoiding any discontinuities of the moments in point A (see Table 1). In both approaches only information from the neighboring elements is used.

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Measurements of Reynolds Analogy for a Hypersonic Turbulent Boundary Layer on a Nonadiabatic Flat Plate

EARL R. KEENER* AND THOMAS E. POLEK*
NASA Ames Research Center, Moffett Field, Calif.

Nomenclature

C_f = local skin friction coefficient, τ_w/q_e
C_h = local Stanton number, $\dot{q}_w/\rho_e V_e(H_{aw}-H_w)$
H = enthalpy
M = Mach number
Pr = Prandtl number
q = dynamic pressure
$\dot{q}$ = heat-transfer rate
r = recovery factor
R_θ = Reynolds number based on boundary-layer momentum thickness
T = temperature
V = velocity
α = angle of attack of test surface to freestream
ρ = density
τ = shear stress

Subscripts
aw = adiabatic wall
e = boundary-layer edge
t = total
w = wall conditions

Received January 17, 1972.

* Research Scientist, Member AIAA.

A COMMON procedure for predicting aerodynamic heating of a surface immersed in a turbulent boundary layer is to use a method for predicting skin friction, together with a Reynolds analogy factor that relates heat transfer to skin friction. In a summary of available information on Reynolds analogy factors for zero-pressure-gradient boundary layers, Cary[1] points out that the often used value of $2C_h/C_f = 1.16$ (recommended in a study by Chi and Spalding) provides a good representation of experimental data for $M \lesssim 5$ and $T_w \approx T_{aw}$. However, for $M \gtrsim 5$, where considerable aerodynamic heating normally occurs, Cary concludes that there are insufficient data and too much scatter in existing data to empirically define the Reynolds analogy factor. Consequently, there is a need for accurate simultaneous measurements of skin friction and heat transfer at hypersonic Mach numbers, especially with conditions of considerable heat transfer.

Seven of the data points included by Cary[1] were preliminary measurements made on a flat plate at Ames and reported by Hopkins et al.[2] Simultaneous measurements of skin friction and heat transfer were made at $T_w/T_{aw} = 0.32$ and $R_\theta = 2600$ to 6200. Not all available data were published and additional measurements were later obtained at higher Reynolds numbers (R_θ up to 18,000). Reynolds analogy factors determined from these additional data from the flat plate test are presented herein.

The experimental investigation was conducted in air in the Ames 3.5 (ft Hypersonic Wind Tunnel, in which cold air is passed through an alumina storage heater system and heated to total temperatures ranging from about 670° to 1170°K. The nozzle was contoured to produce a flow at Mach 7.4. The model was a sharp-edged flat plate, 119 cm long by 43.8 cm wide, mounted on an injection mechanism outside the test section. Thin-skin heat-transfer gages were installed along the centerline of the thick-walled steel plate at 3.18 cm intervals. A skin-friction balance and a boundary-layer Pitot-pressure rake were mounted at 5.1 cm on each side of the centerline at a longitudinal station 100 cm from the leading edge. Measurement of τ_w and $\dot{q}_w$ are estimated to be accurate within 5%. The model was injected into the airstream at angles of attack of 9.3°, 6.2°, 3.1°, 0°, and −2.1°, resulting in Mach numbers at the boundary-layer edge of 5.9, 6.4, 6.9, 7.4 and 7.8, respectively. At each angle of attack the

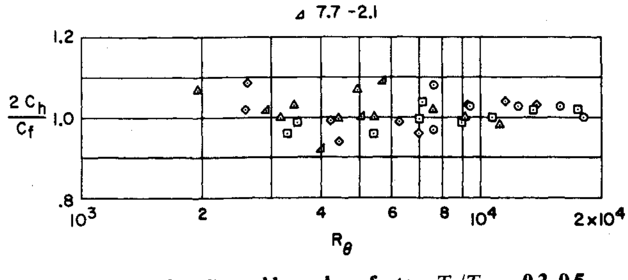

Fig. 1 Flat plate Reynolds analogy factor; T_w/T_{aw} = 0.3–0.5.

local R_θ was varied by changing tunnel stagnation pressure at each of two total temperatures (700° and 1050°K). The resulting range of R_θ was 2,000 to 18,000 and T_w/T_{aw} was about 0.32 and 0.50. The skin-friction results were analyzed and reported by Hopkins et al.[3] who gave additional description of model and test conditions.

The Reynolds analogy factor is defined by the following relationship:

$$2C_h/C_f = V_e(\dot{q}_w/\tau_w)/(H_{aw}-H_w)$$

where

$$H_{aw} = H_e + rV_e^2/2$$

Consequently, the direct determination of $(2C_h/C_f)$ requires the simultaneous measurement of $\dot{q}_w$, τ_w, V_e, T_t, T_w and r, all of which were measured in the present investigation, except the recovery factor r, for which value of $r = 0.88$ was assumed. The use of other values of r for analyzing heat-transfer data is discussed in Ref. 2, however, $r = 0.88$ led to a more satisfactory correlation of heat transfer with skin-friction measurements.

Figure 1 presents the Reynolds analogy factor $(2C_h/C_f)$ as a function of R_θ. There is no distinct effect of Mach number and so all of the data are superimposed but identified by symbols. Maximum scatter is $\pm 9\%$, however, most of the results fall within 4% of $2C_h/C_f = 1.0$ over the large Reynolds number range presented $(2{,}000 < R_\theta < 18{,}000)$.

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Technical Comments

Comments on "Dynamics of an Explosive Reaction Center"

ANDRZEJ WORTMAN*
University of California, Los Angeles, Calif.

ZAJAC and Oppenheim[1] claim to present "the first study of the gas dynamic effects of a reaction center in an explosive gas mixture." This claim is not valid because the authors failed to recognize the principal features of transient spherical and cylindrical flows. Zajac and Oppenheim postulate an impermeable interface between the surroundings and the reaction kernel in which the pressure and temperature are spatially uniform, but time dependent. Chemical reactions of hydrogen with oxygen are computed using 22 forward and backward steps whose rate constants are given by $K = AT^n \exp(-E/T)$ with various constants A, exponents n, and activation energies E for each reaction step. This rather complex chemical model is combined with the ideal gas law in which pressure and temperature are determined from the coupled solutions for uniformly reacting kernel and the ambient gas flow. In effect then, Ref. 1 considers a variation of the classical piston problem in which the expansion is implicitly determined by chemical reactions, rather than specified explicitly.

The expanding kernel generates a primary shock wave (considered by Ref. 1) and also a secondary shock wave (neglected by Ref. 1) which runs into the kernel, causes an order of magnitude spatial variation of temperature and pressure, and thus nullifies the spatial uniformity assumption of Ref. 1 on which all following calculations are based. The complex chemical and characteristics calculations of Ref. 1 refer to a fictitious system which cannot be found in reality.

It is well known (e.g., Stanyukovich,[2] Brode,[3] Friedman,[4] and Glass[5]) that spherical and cylindrical expansions of gases generate an inward-bound expansion fan, followed by a secondary shock wave which points away from the primary shock wave generated by the motion of the contact surface. Essentially, the overexpansion caused by the rarefaction fan and the volume change must be corrected by a shock wave to satisfy the condition of continuous pressure across the contact surface. Initially, the secondary shock wave is weak, but as it approaches the origin, it accelerates and theoretically goes to infinite strength at the origin.[3] An excellent discussion of spherical flows is found in Chapter VI-C of Courant and Friedrichs,[6] where it is pointed out that in the limiting case of strong converging shock waves (Guderley's solution, p. 432), the pressure behind the shock front reflected from the origin is 26 times the pressure behind the incident front. Upon reflection from the origin, the secondary shock wave moves outward and interacts with the interface, thus causing damped oscillations of the system.[2] In a pioneering analytical study of expanding gas spheres, Brode[3] pointed out that the repeated reflections of the secondary shock waves tend to concentrate the energy near the origin in a manner similar to that found in a point source solution. Analyses of such solutions in standard texts (e.g., Sedov[7]) show that pressures and temperatures theoretically go to infinity at the origin.

No exact calculations of the problems considered by Zajac and Oppenheim are presently available for direct comparison with their results. Nevertheless, errors may be assessed using the order of magnitude estimates of the spatial variations of temperature and pressure variations due to the disregarded gas dynamics phenomena. The theoretical limits of infinite temperatures at the origin are not realistic for estimating purposes and it is more appropriate to use existing related experimental data. Glass[5] discusses experimental and theoretical studies of expanding helium and air spheres with initial pressure ratios of 18 and 22, respectively. The surrounding air was initially at the same temperature as the sphere gases. Using the artificial viscosity method like the one employed by Brode, Glass showed that, initially, the pressures and temperatures at the origin decrease by an order of magnitude and subsequently increase by two orders of magnitude when the secondary shock wave is reflected. These results were supported by the experimental data. The pressure ratios in Ref. 5 were higher than those considered by Zajac and Oppenheim (highest pressure ratio 12), but in view of the known theoretical limits, the results obtained by Glass are indicative of the magnitudes of temperature and pressure changes to be expected when the full gas dynamics problem is considered. For a 0.1 mm kernel Figs. 8 and 9 of Ref. 1 show that the temperature rise time and the power pulse width are about 0.1 μsec. With the acoustic velocity of 1 mm/μsec, given in Ref. 1, the time for the initial rarefaction fan to reach the origin is also about 0.1 μsec. Thus, in roughly the characteristic time of the problem an order of magnitude variation of temperature is established

Received July 12, 1971; revision received August 23, 1971.

* Postdoctoral Scholar, School of Engineering and Applied Science; also Engineering Specialist, Aerodynamics Research Branch, Aircraft Division, Northrop Corporation, Hawthorne, Calif.; presently at I.P.P.T.-Z.M.C.iG. Polish Academy of Sciences, Warsaw, on U.S.A. National Academy of Sciences exchange program.